UDC 533. 6.011

ON A PROPERTY OF HEAD-ON COLLISION OF SHOCK WAVES

PMM Vol. 41, № 5, 1977, pp. 951 - 953.

G. M. ARUTIUNIAN
(Moscow)

(Received May 4, 1976)

It is shown that a head-on collision of two shock waves has a simple property which can be utilized in the problems of nonlinear interaction between the shock waves and of their diffraction on moving bodies.

Let two plane shock waves of intensities p_1 and p_2 (Fig. 1) propagate towards each other from the opposite directions through an unperturbed medium. In the pV-plane the waves have the corresponding points *1* and *2* ($V = 1 / \rho$ denotes the specific volume of the gas) on the shock adiabate (curve a in Fig. 2) emerging from the point O corresponding to the unperturbed state of the gas.

Let us project, from the points *1* and *2* in the pV-plane, two new shock adiabates (curves b and c in Fig. 2). Their equations are

$$\frac{V}{V_i} = \frac{(\gamma+1)p_i + (\gamma-1)p}{(\gamma-1)p_i + (\gamma+1)p}, \quad \frac{V_i}{V_0} = \frac{(\gamma+1)p_0 + (\gamma-1)p_i}{(\gamma-1)p_0 + (\gamma+1)p_i} \quad (i = 1, 2) \tag{1}$$

Eliminating the variable V we arrive, at $p_1 \neq p_2$ to a quadratic equation for the ordinate of the point of intersection, and solving this equation for p we obtain

$$p = p_1 p_2 / p_0 = p_*, \quad p = p_0 \tag{2}$$

The corresponding abscissas are determined from (1). The second point of intersection (p_0, V_0) has no physical meaning. Consequently, if $p_1 \neq p_2$ another two shock waves of equal intensity propagate along the shock waves *1* and *2* (Fig. 1), and the densities behind two new shock waves will be equal provided that their intensities are equal to p_*.

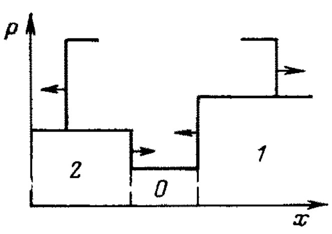

Fig. 1

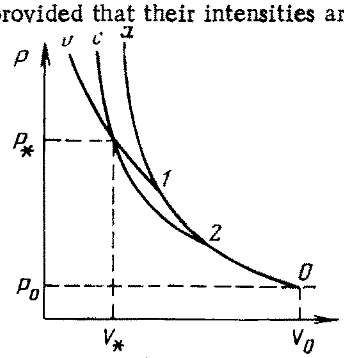

Fig. 2

We know that a head-on collision of the shock waves *1* and *2* generates two shock waves of equal intensity propagating along these waves and separated from each other by the surface of contact discontinuity (Fig. 3). We shall show that, irrespective of the values of p_1 and p_2 the following relation holds for the resultant pressure p_r

$$p_r \leqslant p_1 p_2 / p_0 \tag{3}$$

where the equal sign applies only when $p_1 = p_0$ or $p_2 = p_0$.

Let us define by the conditions $p_1 \geqslant p_0$, $p_2 \geqslant p_0$ a region in D in the p_1p_2-plane, and consider in this region a function ζ defined as follows:

$$\zeta(p_1, p_2) = p_r(p_1, p_2) - p_1 p_2 / p_0 \tag{4}$$

Clearly, at the boundaries of the region D we have

$$\zeta(p_1, p_0) = \zeta(p_0, p_2) = 0 \tag{5}$$

$$(p_r(p_1, p_0) = p_1, \; p_r(p_0, p_2) = p_2)$$

In the part of D where $p_1 > p_2 > p_0$, we have $\zeta \neq 0$. Indeed, if $\zeta \neq 0$ at any point of this part of D, then (4) would imply that

$$p_r(p_1, p_2) = p_* \tag{6}$$

and this is impossible since when shock waves of differing intensities collide head-on (see Fig. 3), the gas densities on both sides of the contact discontinuity must differ from each other and this would contradict (by virtue of the above argument) the relation (6).

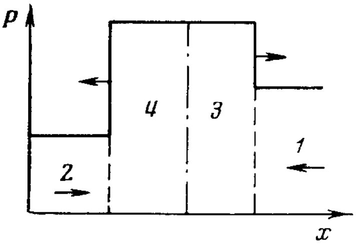

Fig. 3

Using similar argumentation, or simply by symmetry considerations (since $p_r(p_1, p_2) = p_r(p_2, p_1)$, $\zeta(p_1, p_2) = \zeta(p_2, p_1)$) we con-conclude that $\zeta \neq 0$ also when $p_2 > p_1 > p_0$. The case of $p_2 = p_1$ corresponds to head-on collision of shock waves of equal intensities, therefore (4) yields, after simple transformations

$$\zeta(p_1, p_1) = -\frac{(\gamma - 1)(p_1 - p_0)^2 p_1}{[(\gamma - 1) p_1 + (\gamma + 1) p_0] p_0} < 0 \tag{7}$$

$$(p_1 > p_0)$$

Thus $\zeta = 0$ within the whole region D with the exception of its boundary.

Taking into account the continuity of $\zeta(p_1, p_2)$ (which follows from the continuity of p_r), we can deduce from (7) that $\zeta(p_1, p_2) < 0$ when $p_1 > p_0$ and $p_2 > p_0$. This, together with (5), yields the relation (3).

Let us consider some corollaries of the property (3) of any head-on collision of shock waves. When $p < p_*$ the shock abiabate b in Fig. 2 passes to the right of the shock abiabate c, and by virtue of (3) we have $p_r < p_*$. It follows therefore that when the shock waves undergo a head-on collision, the resulting density will be higher at the side of the contact discontinuity (Fig. 3) from which the weaker of the colliding waves arrives. The relations between the resulting temperatures will obviously be opposite.

From what has been said it follows that, in particular, if a shock wave impinges on a body moving at supersonic speeds, then after the first reflection from the surface of the body, it will be reflected from the surface once again provided that the intensity of the impinging wave is less than the intensity of the bow shock wave. This can be explained by the fact that in view of what has been said $\rho_4 > \rho_3$ (Fig. 3).

The problem of secondary reflections was studied earlier in [1].

The author thanks E. I. Ruzavin, S. K. Shimarev and E. G. Shifrin for assessment of the results and for interest shown.

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Translated by L. K.